

# Sequential Decision Rules for Managing Nematodes with Crop Rotations

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**Abstract:** A dynamic model of nematode populations under a crop rotation that includes both host and nonhost crops is developed and used to conceptualize the problem of economic control. The steady state of the dynamic system is used to devise an approximately optimal decision policy, which is then applied to cyst nematode (*Heterodera schachtii*) control in a rotation of sugarbeet with nonhost crops. Long-run economic returns from this approximately optimal decision rule are compared with results from solution of the exact dynamic optimization model. The simple decision rule based on the steady state provides long-run average returns that are similar to the fully optimal solution. For sugarbeet and *H. schachtii*, the simplified rule can be calculated by maximizing a relatively simple algebraic expression with respect to the number of years in the sequence of nonhost crops. Maximization is easy because only integers are of interest and the number of years in nonhost crops is typically small. Solution of this problem indirectly yields an approximation to the optimal dynamic economic threshold density of nematodes in the soil. The decision rule requires knowledge of annual nematode population change under host and nonhost crops, and the relationship between crop yield and nematode population density.

**Key words:** crop rotation, cyst nematode, decision rule, dynamic control, economics, economic threshold, *Heterodera schachtii*, optimization, sugarbeet.

The concept of an economic threshold population at which a pest management practice becomes marginally profitable is well defined and easily understood for direct control measures (usually pesticides) if consequences of the action are irrelevant for future control actions, either with respect to the present crop or subsequent crops. Therefore, the term, as traditionally defined (12), is intrinsically a static concept used in its simplest form where the time of pesticide application is predetermined, only one possible application level is considered, and the decision is based on the estimated pest population at the specified time of application. An early generalization was to choose both the critical population level for application jointly with the time for application (10). Talpaz and Borosh (17) extended the model further by introducing multiple application times and rates within the same season; they also provided for a setup cost of application in

addition to the cost of pesticide. However, severe restrictions were placed on the set of alternative pest population control measures to reduce the optimization problem to only two variables. Nevertheless, these generalizations still require that there are no dynamic effects from pest control measures this year on subsequent years, and tacitly assume no measurement error on the pest population.

The application of economic threshold concepts to soil-dwelling plant-parasitic nematodes involves some unique considerations. The pest population is already present in the field in measurable quantities at the time the crop is planted, and there is a relationship (defined with varied precision) between the number of nematodes present and expected crop damage. Also, the management decision is usually made before time of planting (8). Year-to-year dependencies in nematode population levels at a given location can be of great importance; that is, the number of nematodes of a given species that will be present in the soil next year is determined by management decisions that are made to avoid crop damage this year. At best then, as recognized in the literature (4,8), application of the static concept of economic threshold is of limited usefulness. How-

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ever, such a short-run criterion can be informative, and Ferris (4) defined an annual profit function for a model where the nematode population and control variable were continuous, and then derived the necessary conditions for profit to be a maximum. These results could be useful as a short-run approximation when using a chemical control where it is thought that multiyear effects of the control on the pest population are relatively small.

Ferris (4) extended the economic trade-off equation represented by the necessary conditions in the static maximization problem for the case where a crop rotation, instead of a nematicide, is used to control nematodes. His sequential decision rule for choosing a crop uses as a demarcation point the nematode population where returns from the host and nonhost crops are equal, which we denote  $N^*$ . In its most rigorous interpretation, if the population is less than or equal to  $N^*$ , the host crop is planted; if greater than  $N^*$ , the nonhost crop is planted. If this rule could be applied on a continuous-time basis for control of the nematode population (i.e., at fractions of a year), the population would be shifted to a point where returns in the host and nonhost crops were equal. Then the relatively high return of the host crop in the absence of high nematode densities would be reduced to equal that of the nonhost crop—clearly an uneconomic result because the primary purpose for having the host crop in the rotation would be gone. In the discrete choice situation, the population is moved so that it is lower than the obviously uneconomic continuous variable solution. The logical contradiction of the requirement that nonhost crops should be grown longer than indicated by the decision rule, in order to reduce the nematode population below  $N^*$  and permit a surplus to be realized from the relatively high return host crop, is problematic. This static model cannot be used to quantify the optimal *dynamic* economic threshold that would be characterized by a smaller population than  $N^*$  obtained by Ferris' rule. A

conceptual framework is needed to choose the rotation for which the present value of profits from a given acreage is maximized.

## THEORY

*Dynamic behavior of nematode populations:* The simplest cropping sequence for management of nematode damage is one in which the host crop is grown for only 1 year, after which nonhost crops are planted for 1 or more consecutive years, and then the host crop is again planted for a single year. An example would be sugarbeet when the soil is infested with cyst nematodes (*Heterodera schachtii*) in a mild climate such as the Sacramento Valley in California. The following notation is introduced:

- P = a quantitative measure of pest density in the soil measured at planting time for the host crop (commonly denoted  $P_i$  in the literature)
- s = the proportion of the pathogen that survives during a year of nonhost crop
- $R_{at}$  = annual returns from the nonhost crops, where t denotes the consecutive year after the host crop
- M = the number of years that nonhost crops are grown in succession
- $R_h(P)$  = returns from the host crop if P is the pathogen density at planting.

Slope of the function  $R_h(P)$  is negative because the pathogen depresses crop yield. It is frequently obtained via the relationship  $y = (m + (1 - m)z^{(P-T)})$ , where y is yield expressed as a proportion of the expected yield in the absence of nematodes, m is the minimum yield attainable at high nematode levels, z is a regression coefficient  $< 1.0$ , and T is the tolerance level or nematode population level below which there is no reduction in yield (3,4,8,15).  $R_h(P)$  is obtained by subtracting production costs from the crop value of y. If a nonhost

crop is planted in year  $t$ , the population dynamics are described by

$$P_{t+1} = sP_t, \quad (1)$$

and for a year  $t$  in which the host crop is planted,

$$P_{t+1} = g(P_t), \quad (2)$$

where the function  $g(P)$  measures net growth in the nematode population density for the full year. The function  $g(P)$  is expected to be concave from below with positive slope, except at high population densities where the slope might be negative (16). Successive substitution into Eq. [1], starting with  $P_0$  to calculate  $P_1$ , demonstrates that after  $M$  years  $P_M = s^M P_0$ .

A rotation cycle is defined as the  $(M + 1)$ -year period that begins with the host crop and is followed by  $M$  years of nonhost crops. Letting  $n$  denote a particular rotation cycle that begins with planting the host crop under a population density  $P(n)$ , the population is transformed into  $g(P(n))$  during that year according to Eq. [2]. Then during the  $M$  years in the nonhost crop sequence that follow, the population is transformed further according to Eq. [1] to yield a general recursive relationship among the rotation cycles,

$$P(n+1) = s^M g(P(n)). \quad (3)$$

The relationship in Eq. [3] is a first-order difference equation, where  $M$  is simply a parameter in the general functional form of the right-hand side. Samuelson (14) summarizes the mathematical properties of this type of equation. For a concave functional form that assumes only positive values, the steady state is approached asymptotically from any arbitrary initial condition. In this application, that initial condition is the nematode density at the time the host crop is planted, and the associated steady state is denoted  $P_M^*$ . It is important to recognize that the steady-state nematode density is independent of the initial condition, say  $P_0$ , and  $P_M^*$  can be found by solving the equation,

$$P_M^* = s^M g(P_M^*), \quad (4)$$

which is Eq. [3] with the same variable on both sides.

The nematode population is at the level  $P_M^*$  at only a single point in the rotation cycle, viz., planting time for the host crop, and there exists an associated steady state for each of the other  $M$  points within the rotation cycle, one for each year in the sequence of nonhost crops in the rotation. With the process in steady state, the population is transformed from  $P_M^*$  to  $g(P_M^*)$  during the year that the host crop is grown, and after a subsequent period of  $t$  years in nonhost crops, the population would be  $s^t g(P_M^*)$ ,  $t = 1, 2, \dots, M$ . The host crop is planted at the beginning of year  $M + 1$  and  $P_M^*$  is reproduced via Eq. [3], thus completing the steady-state cycle. Consequently, the population dynamics involve an entire schedule of  $M + 1$  population measures throughout the rotation cycle, and it is this schedule that converges to a steady state, starting from an initial nematode density at any particular year within the cycle. In the above example, the year that the host crop is planted has been used in the discussion as the reference point for beginning a new cycle.

From the above discussion and notation, the steady-state value  $P_M^*$  is unique to a specific number of years in the nonhost crop sequence of the rotation, i.e.,  $M$  years. The correspondence between  $P_M^*$  and  $M$  exists in the steady state, but starting from an arbitrary state of the process, it can be controlled by focusing on either  $M$  or  $P_M^*$ . Using  $M$  is intuitively appealing in that the fixed rotation over time is initiated as quickly as the existing cropping history will permit. In contrast, the steady-state value of  $P$  is used as a threshold such that the host crop is planted whenever  $P$  is less than or equal to  $P_M^*$ . Although either method of control will yield the same steady state, the time paths in general will be different when starting from an arbitrary level of  $P$ . While the transition path using  $M$  will always have cycles of fixed

length regardless of the initial value of  $P$ , that will not necessarily be the case when using  $P_M^*$  as a decision variable. A decision rule based on  $P_M^*$  is preferable because it focuses on the object of control, viz., the nematode population at the time the host crop is planted.

The dynamic behavior of nematode population density cycles when  $P_M^*$  is used to determine when to plant the host crop is illustrated in Fig. 1, which was constructed from data (3,11) in an application presented later. The bars indicate population densities of *H. schachtii* at planting time for the sugarbeets regardless of what crop is planted in a given year. The initial density before planting sugarbeets is 10 eggs/100 g soil (hardly perceptible as the first bar of the graph); then the population increases to 1,170 and slowly erodes to 150 during the next 5 years of nonhost crops (annual survival rate  $s = 0.667$ ). The second cycle has a peak density of 3,220 at planting time the year after the sugarbeet crop and then erodes to 190 during 7 years of nonhost crops. Convergent population densities for the year before the host crop

is planted and the year after are 210 and 3,610, respectively. Convergence of the entire schedule of  $M+1$  population measures is illustrated by scanning the graph in Fig. 1 from left to right.

A simplified decision framework is proposed that uses the number of years in nonhost crops,  $M$ , as the choice variable and indirectly determines the steady-state value  $P_M^*$  defined above for the difference equation in Eq. [3]. But crop planting intervals constitute a discontinuous time scale, so this model is limited to an integer choice variable,  $M$ , with no direct attention given to the currently observed pest density,  $P$ , which is a continuous variable. However, results from the steady-state optimization with respect to  $M$  are reinterpreted to allow a sequential decision framework using  $P_M^*$  as the dynamic economic threshold for planting the host crop. Empirical evidence presented later demonstrates that using the integer choice variable provides an excellent approximation of full-fledged dynamic optimization. A flexible decision rule using  $P_M^*$  as an economic threshold is particularly advan-

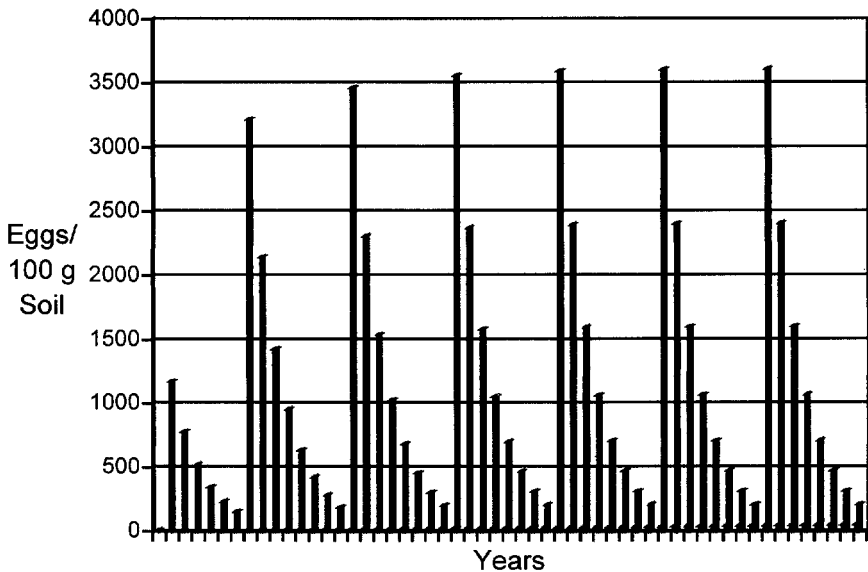


FIG. 1. Population density cycles of *Heterodera schachtii* where the host crop (sugarbeet) is grown for a single year and followed by consecutive years in nonhost crops until the nematode population density falls below 225 eggs/100 g soil, at which time the host crop is planted again, thus starting a new sequence of host and nonhost crops and a new population cycle. Note that the first population cycle occurred in 7 years and the remainder in 8 years.

tageous when year-to-year changes in population densities are recognized as a random variable subject to climatic variations.

*A simple pedagogical decision model:* We begin with the criterion of *maximization of average annual returns* from the rotation cycle. This simplified criterion does not consider the importance of compound interest on cash flows of the business. A more complex economic criterion, further developed later, would maximize a weighted average of returns where the weights involve the interest rate and distance in time from when the returns are received. The simpler criterion can be used to derive an economic trade-off equation that builds intuition for the more general case. The focus is on the rotation that would be approached when an optimal decision rule is applied sequentially from an arbitrary initial condition. This steady-state decision rule is obtained by maximizing

$$F(M,P) = [R_{a1} + R_{a2} + \dots + R_{aM} + R_h(P)] / (M + 1) \tag{5}$$

with respect to P and M, subject to the constraint

$$P = s^M g(P), \tag{6}$$

which is essentially Eq. [4] to force the steady state. The solution values for P and M satisfy the definition of  $P_M(*)$  in the notation defined above. Numerical solution is fairly simple in that an integer is chosen for M and then Eq. [6] is solved for the one unknown, P. This value is then substituted into Eq. [5] to compute the criterion function, F(M,P). The maximum can be obtained by a direct search on M and tabulation of the values of F(M,P). This search yields the dual values, M and  $P_M(*)$ , which are associated with the constrained maximization problem in Eqs. [5] and [6]. As explained later, it is  $P_M(*)$  that is of primary importance for making practical economic decisions if nematode density can be estimated in the field with sufficient accuracy at a nominal cost.

Ferris (7) suggests that a double-log-linear model adequately describes both

nematode multiplication rates during the growing season and overwinter survivorship, although an exception might be necessary for large initial nematode populations in sugarbeets to allow for an interval where the response function has negative slope (3). Embedding the overwinter survivorship function in the argument of the seasonal multiplication rate function provides the annual population function  $g(\cdot)$  in Eq. [6] with the same form. Letting a and b be parameters with  $a > 0$  and  $0 < b < 1$ , gives the explicit function

$$g(P) = aP^b. \tag{7}$$

The steady-state equation [6], is  $P = s^M (aP^b)$ , can be solved explicitly for P as a function of M to get

$$P = (as^M)^{1/(1-b)} = cd^M. \tag{8}$$

The two parameters c and d are a and s, respectively, taken to the power  $1/(1 - b)$ . This explicit expression for P in Eq. [8] simplifies numerical computation of the criterion function for a given value of M because P in the right side of Eq. [5] can be replaced with  $cd^M$ . Since  $0 < b < 1$ , it follows that  $d < s$ . Thus d reflects the overall annual impact that an extension of the nonhost crop period has on the steady-state nematode population, which includes the reduction phase (nonhost crop) and increment phase (host crop). The functional form in Eq. [8] implies that if  $P_M(*)$  is the steady state associated with M years in the nonhost crop, then for a cycle of  $M + 1$  years

$$P_{M+1}(*) = cd^{M+1} = d(cd^M) = dP_M(*). \tag{9}$$

To obtain a short-run decision rule that can be applied as an approximately optimal rule in an operational setting where the year-to-year population dynamics involve random elements, we derive an inequality for practical application. Although it might be impossible to obtain an explicit solution for P as a function of M in Eq. [6] for some functional forms, the function always exists conceptually, and it is denoted  $f(M)$ . An example of  $f(M)$  is Eq. [8] for the special functional form of  $g(P)$

in Eq. [7]. Substitution of  $f(M)$  for  $P$  in  $F(M,P)$  in Eq. [5] gives an equation in the single variable  $M$ , which we denote  $h(M)$ . For a function that has more than one local maximum, the following condition is necessary for a *local* maximum:  $h(M-1) \leq h(M) \geq h(M+1)$ . Under the assumption that  $h(M)$  is unimodal,  $M$  maximizes  $h(M)$  if the following inequalities hold,

$$h(M) \geq h(M+1) \text{ and } h(t) \leq h(t+1), t = 1, 2, \dots, M-1.$$

Starting with  $h(M) \geq h(M+1)$ , multiplying both sides by  $M+2$ , and making a few algebraic manipulations yields

$$R_h(f(M+1)) - R_h(f(M)) + R_{a,M+1} \leq [R_{a1} + R_{a2} + \dots + R_{aM} + R_h(f(M))]/(M+1). \quad (10)$$

Note that the right side of the inequality in Eq. [10] is the maximized value of the criterion function in Eq. [5] and the left side is the increment in returns that would be realized by delaying the planting of the host crop 1 more year. The first two terms on the left measure the value of yield enhancement from a 1-year delay in planting the host crop, and the third term  $R_{a,M+1}$  is the direct return from the nonhost crop.

Letting  $g(P)$  take the special functional form in Eq. [7],  $f(M+1)$  in Eq. [10] can be replaced by an explicit expression in  $P$ . If  $P$  is the nematode population this year ( $M$ ), then using Eq. [9] we see that the population next year ( $M+1$ ) after another year of nonhost crops is  $dP$ . It is convenient to use this notation for the general case and rewrite Eq. [10] for practical interpretation as

$$R_h(dP_e) - R_h(P_e) + R_{a,k+1} \leq [R_{a1} + R_{a2} + \dots + R_{aM} + R_h(f(M))]/(M+1), \quad (11)$$

where  $P_e$  on the left denotes the estimated or measured population density at planting time for the host crop and  $k$  is the number of years that the nonhost crops have already been grown without interruption; while  $f(M)$  on the right side denotes the steady-state population density associated with the optimal long-run rota-

tion length denoted  $M$ . If the inequality is met, the host crop is grown; if not, the nonhost crop sequence is continued. With this interpretation, we can think of the right side of Eq. [11] as optimized average annual returns under long-run average conditions and a steady state for the nematode population. But the left side is viewed as a measure of the short-run benefits from a delay in planting the host crop for 1 more year, where the net return measures reflect a best practical estimate of the field population density of nematodes, and could also be adjusted for relative crop prices that are expected for the two crops this year compared to next. This rudimentary decision rule is relatively simple and can be explained to managers because its structure is that of comparative marginal analysis that appeals to intuition.

From Eqs. [5] and [6], a generalization of the model can be formulated to accommodate 2 or more consecutive years in the host crop. The second year in the host crop would have a return equal to  $R_h(g(P))$ , which would be an added term in Eq. [5]. The steady-state equation [6] would be changed to  $P = s^M g(g(P))$  because growth in the nematode population would need to reflect two consecutive years of applying the growth function  $g(\cdot)$  to the starting population  $P$ . For the case where  $g(P)$  is given by Eq. [7],  $P = s^M a(aP^b)^b$  and the explicit solution for  $P$  yields a 2-year version of Eq. [8], i.e.,  $P = a^{1/(1-b)}(s^{1/(1-b^2)})^M$ . The search on  $M$  for the maximum steady-state solution would compare the maximized values of  $F(M,P)$  under the 1- and 2-year regimes for the host crop, a doubling of the computations but still a very simple task, particularly if the specific functional form of Eq. [7] is used since it gives  $P$  explicitly in terms of  $M$ .

*An economic model with discounting:* The logical framework and notation are the same as above except that *cash flows associated with the crops are discounted to a common point in time* to account for the time value of money, i.e., the fact that cash can be invested at compound interest or used to pay off debt and save interest payments. If  $r$  is

the annual interest rate, then  $(1 + r)$  is the *compounding factor* and  $w$  denotes its inverse,  $1/(1 + r)$ , which is called the *discount factor*. The average annual measure of returns in Eq. [5] is replaced by a weighted average where the weights are  $1/(1 + r)^t = w^t$ , for distance  $t$  in the future. The sum of these weights for  $t = 1, 2, \dots, n$  is  $[1 - (1 + r)^{-n}]/r$ . Therefore, the weighted average return (annuity) from an  $M + 1$  year rotation comparable to Eq. [5] is given by

$$[R_{a1}w + R_{a2}w^2 + \dots + R_{aM}w^M + R_h(P)w^{M+1}] \frac{r}{1 - (1 + r)^{-(M+1)}} \quad (12)$$

where the discrete time periods are defined so that annual returns are received at the end of the year. The second equation that works with Eq. [12] to provide  $P$  as a function of  $M$  is again Eq. [6] as in the simple unweighted returns case (Eq. [5]). The optimal length steady-state constrained rotation is obtained by searching for the maximum annuity over integer values of  $M$  by first computing  $P$  from Eq. [6] for given  $M$ , and then computing Eq. [12]. Of course, the computations are simplified by using Eq. [7] as a special case for Eq. [6] because  $P$  in Eq. [12] can be replaced by  $cd^M$  from Eq. [8]. In the general notation of the last section,  $P$  in Eq. [12] is equal to  $f(M)$ .

An inequality analogous to Eq. [10] can be derived by comparable but more tedious algebraic manipulations, but these are relegated to Appendix A. Since Eq. [11] is a form that is more intuitively applied as a sequential decision rule, its analogue with discounting is presented,

$$[R_h(dP_e) - (1 + r)R_h(P_e)] + (1 + r)R_{a,k+1} \leq [R_{a1}w + R_{a2}w^2 + \dots + R_{aM}w^M + R_h(f(M))w^{M+1}] \frac{r}{1 - (1 + r)^{-(M+1)}} \quad (13)$$

The population density of the nematode in the field at planting time for the host crop, determined by sampling, is  $P_e$ , and the eco-

nomic trade-off associated with the inequality is focused on the point in time 1 year hence. The right side measures the reward from planting the host crop, while the left side measures the reward from planting the nonhost crop instead and waiting until next year to reconsider planting the host crop. The right side of Eq. [13] is the constant return per year that would be equal in present value to the infinite stream of irregular returns associated with an optimal rotation beginning immediately after the host crop has been grown. The values of  $M$  and  $P$  are those for the optimal steady-state rotation and must satisfy Eq. [6]. In the square brackets on the left side of Eq. [13] is the positive difference in returns from the host crop when the nonhost crop sequence is grown 1 more year, and the returns from the shorter rotation are compounded forward 1 year to account for those returns being received 1 year earlier. Returns for the nonhost crop in its  $(k + 1)$ -year in succession are also compounded forward 1 year for the same reason.

The smallest value of  $M$  such that Eq. [13] holds is a necessary condition for the expression in Eq. [12] to be a maximum, provided it is unimodal with respect to  $M$ , a condition that should be verified in applications. The steps in applying Eq. [13] would be to first calculate the right side for  $M = 1, 2, \dots, M_{max}$ , where  $M_{max}$  is a practical upper bound for the optimal value  $M^*$ , which maximizes Eq. [12] with respect to  $M$ . This would determine the optimal rotation in a nonrandom environment, but random fluctuations in the rate of decline in  $P$  under nonhost crops, and the rate of growth of  $P$  during the year in a host crop, make it advantageous to use the inequality in Eq. [13]. The decision agent should think of the right side as a predicted value of future annual returns, while the left side is a dynamic measure of estimated immediate returns in light of current evidence on the level of the nematode population. The immediate return is couched in a marginal framework for comparison to expected long-run average returns. Al-

though not explicitly taken into account in the discussion, there is also the uncertainty of the parameters and functional form of the technical relationships that quantitatively describe the dynamics of the nematode population under both host and non-host crops. This is an added reason for field sample measures of the nematode population density at planting time for the host crop.

The economic model with discounting can be generalized to allow 2 or more years in the host crop. For the necessary adjustments, refer to the discussion of the model without discounting in the previous section. So far the stream of returns from the nonhost crops has been taken as given, but often there is considerable flexibility without compromising good agronomic practices. Where technical constraints are no problem, economic considerations can be used to optimize the sequence of crops. When one of the nonhost crops is a perennial like alfalfa hay that has first an increasing and then decreasing annual return from a new stand, this crop should be placed in the sequence where it will be involved in the marginal decision of how long to stay in nonhost crops before replanting the host crop. This is so because it allows optimal choice of the length of the alfalfa stand jointly with optimization of the period in all nonhost crops, but the analyst must allow for shortening the rotation by either terminating the alfalfa stand a year early or deleting an annual crop from the sequence ahead of the alfalfa. However, a preliminary analysis of the economic viability of alfalfa in the rotation should be done by choosing stand length to maximize weighted average net returns from alfalfa; if this is not competitive with some other crops that could be equally effective in the rotation, then alfalfa should not be grown. The overall economic principle is to choose the sequence to maximize amortized present value of returns from any given length of the sequence in non-host crops, with appropriate recognition to the economic value of synergistic relation-

ships among the crops in any particular sequence.

Averages of net returns calculated for perennial crops should be weighted averages that employ the discount factor. Since these calculations are projected budgets into the future, the discount rate should be the *real* rate as opposed to the nominal rate. Assuming no risk of default by the borrower, the nominal rate is the sum of two components: (a) the real rate that tends to be rather constant around 4% to 5%, and (b) the expected rate of inflation that is subject to much uncertainty. The latter component is necessary in commercial contracts, which are in nominal dollars, because loans are paid back with dollars that have less purchasing power than those that were lent. But in crop budgeting, we are using dollars with constant purchasing power, thus the reason for using the lower real discount rate. Consequently, the interest rate  $r$  introduced above should be interpreted as the real rate.

*A fully dynamic economic model:* A more general economic decision framework is provided by focusing on the sequentially observed nematode density,  $P$ , instead of the period in nonhost crops,  $M$ , as the decision variable. In the simple case where returns are the same for each year of the nonhost crop, the currently observed nematode population density summarizes the entire history of the process for economic decision purposes, i.e., any other detail would be redundant. (More general cases will be discussed in the application below.) Under this fully dynamic economic criterion, a *decision rule* is defined that specifies whether the host or nonhost crop is planted after observing a value of  $P$  (1). This rule indirectly controls lengths of the rotation cycles and, starting from an initial value  $P_0$ , a sequence of integers  $M_1, M_2, \dots$  would be implied instead of a single value of  $M$  as in the above discussion. The focus is entirely on pest population density,  $P$ , and convergence is with respect to this continuous variable instead of the integer  $M$ . Intuitively, convergence takes



place because we are working with an infinite planning horizon, and at the end of each year after observing a given value of  $P$ , the optimization problem is exactly the same for that given value of  $P$  because none of the functional relationships are specified to depend on time per se.

Using a sequential optimization model that derives a decision rule as a function of  $P$ , the associated steady state can involve two or more cycles of host and nonhost crops of different lengths, thus forming the overall aggregate steady-state cycle. For example, the latter could be comprised of  $M_1$  and  $M_2$  years of nonhost crops with the intervening host crop after each, making an aggregate cycle of  $M_1 + M_2 + 2$  years. This phenomenon occurs because  $P$  is continuous while  $M$  must be an integer. Therefore, the above steady-state methodology might not find the overall best critical value for  $P$  because the steady state is limited to a single cycle through the host and nonhost crops.

## RESULTS

Necessary empirical measures for application of the economic model to actual management decisions for *H. schachtii* and sugarbeet are not readily available, but enough information is available in the literature to provide examples of results that can be expected in practice. The discount rate is assumed to be 5%. Specific quantitative measures at a given location are needed for  $s$  in Eq. [1]; the function  $g(P)$  in Eq. [2] (e.g., the parameters  $a$  and  $b$  in Eq. [7]); the return function  $R_h(P)$ , which traditionally involves a Seinhorst-type yield function with a damage tolerance level (3,15); and finally, returns from the nonhost crop sequence.

The biological relationships summarized by Eqs. [1] and [2], and sugarbeet yield response to population levels of the sugarbeet cyst nematode that determines  $R_h(P)$ , are based on Cooke and Thomason (3); gross margins (returns above variable costs) for nonhost crops are taken from

Malarin (11). The former are for the Imperial Valley and the latter the Sacramento Valley in California, so they are not compatible, but results are calculated for a range of parameters in the relationships involving population dynamics to evaluate the sensitivity of results. Warmer soil temperatures in the Imperial Valley allow more generations of nematodes during the year of sugarbeet (affecting  $g(P)$ ) and a more rapid decline in viable eggs during years of nonhost crops (affecting  $s$ ); the warmer temperatures could also affect the sugarbeet yield response function, and thus  $R_h(P)$ . It is easy to provide results for various values of  $s$ , but  $g(P)$  was more problematic.

The empirically estimated equations are:

$$R_h(P) = 1,188(.99886)^{P-100} - 627, P > 100, \quad (14)$$

$$R_h(P) = 561, P \leq 100, \\ g(P) = 501.4P^{0.3689}. \quad (15)$$

Subtraction of 100 from  $P$  in  $R_h(P)$  in Eq. [14] implies a tolerance level ( $T$ ) equal to 100, where  $P$  is number of eggs/100 g of soil. The coefficient, 1,188, is the product of maximum yield (27 tons/acre) and price(\$44/ton), and 627 is variable costs of production per acre. A total of four nonhost crops were considered in various models with the following estimated gross margins per acre: alfalfa \$210, corn \$383, tomatoes \$746, and wheat \$279 (11).

The parameters in Eq. [15] for  $g(P)$  were estimated with the three observations contained in Table 2 in Cooke and Thomason (3), where each data point is an aggregate over several plots. A weighted linear regression in logarithms of the variables that accounted for the relative number of plots was used. This gave relatively little weight to the third observation with largest initial population, which exhibited negative marginal response. This functional form forces a monotone increasing function. As an alternative parameterization, the third observation was deleted with the result,

$g(P) = 266.0P^{.5324}$ . Both of these equations predict rather large populations the next year for quite small values of P, if one were considering application to the Sacramento Valley. Therefore, the above formula in Eq. [15] was modified by dividing the constant term by 2 to give

$$g(P) = 250.7P^{0.3689} \quad (16)$$

for the Sacramento Valley. Values of *s* from 0.50 to 0.75 were used.

Note that the steady-state equation [6] is the special case in Eq. [7] for the above specification, and the explicit expression for P in Eq. [8] is

$$P = 6,332s^{1.5845M}, \quad (17)$$

when applied to Eq. [16]. A great deal can be learned about the proximity of the economically optimal rotation length by simply tabulating the values of P for given values of M in Eq. [17], as was done to construct Table 1. The smallest rotation length for which the population density drops below the tolerance level of 100 is footnoted, thus showing the important effect that *s* has on likely economic rotations. The first row of Table 1 where M = 1 gives the density at the beginning of the first year after sugarbeets have been

grown when the cropping system is in steady state.

*Results for the steady-state economic model:* Results of applying the economic model represented by Eqs. [12] and [13] when returns from nonhost crops are constant at \$210 (an average for alfalfa) are given in Table 2, columns 4 and 5, with the nematode rate of survival, *s*, equal to 0.667. The maximum annual equivalent returns are \$247, which is attained with 6 years in nonhost crops. The fifth column in the table labeled Delta(M) is the left side of Eq. [13], which is a measure of marginal returns from delaying the planting of sugarbeets 1 more year. Notice that this value exceeds the annuity (weighted average returns) when *M* < 6 and is less than the annuity when *M* ≥ 6, which is the necessary condition for *M* = 6 to yield maximum returns. Results in columns 6 and 7 are for the same situation except returns in nonhost crops are \$470 and the optimal period in nonhost crops is 7 years.

Results for an irregular stream of returns from nonhost crops that average \$470 are given in columns 8 and 9 with all other parameters the same. (The sequence is tomato, corn, and wheat.) In this case the optimum is 7 years in nonhost crops, and examination of the last column for Del-

TABLE 1. Steady-state population densities of *Heterodera schachtii* ( $P_M^{(*)}$ ) in relation to annual proportional survival (*s*) during a nonhost crop and the number of years that nonhost crops are grown in succession (*M*), derived from the model  $P = (as^M)^{1/(1-b)}$  where *a* and *b* are parameters describing annual changes in the nematode population (Eq. [8]).

Years (M)	Steady-state densities of <i>Heterodera schachtii</i> ( $P_M^{(*)}$ , eggs/100 g soil)				
	Annual proportional survival during nonhost crop ( <i>s</i> )				
	0.500	0.600	0.667	0.700	0.750
1	2,111	2,819	3,331	3,598	4,014
2	704	1,255	1,752	2,045	2,545
3	235	558	921	1,162	1,613
4	78 <sup>a</sup>	249	485	660	1,023
5	26	111	255	375	648
6	9	49 <sup>a</sup>	134	213	411
7	3	22	71 <sup>a</sup>	121	260
8	1	10	37	69 <sup>a</sup>	165
9		4	20	39	105
10		2	10	22	66 <sup>a</sup>

<sup>a</sup> Smallest M for which egg density is below the tolerance level (T) of 100 eggs/100 g soil.

TABLE 2. Weighted average returns determined by a steady-state search, using the model represented by Eqs. [12] and [13], for the number of years necessary in crops that are nonhosts to *Heterodera schachtii* and have constant net returns (A and B), or differing net returns, e.g., tomato, corn, and wheat (C). Annual proportional survival (*s*) under a nonhost crop assumed to be 0.667.

Years (M)	Density (/100g)	Host returns (\$)	Net returns from nonhost crops					
			A \$210		B \$470		C \$746, 383, 279,...	
			Annuity (\$)	Delta(M) (\$)	Annuity (\$)	Delta(M) (\$)	Annuity (\$)	Delta(M) (\$)
1	3,333	-597	-184	401	-51	674	91	582
2	1,755	-447	2	527	179	800	247	600
3	924	-163	124	529	323	802	329	1,092
4	486	138	197	443	410	716	467	625
5	256	367	233	350	455	623	490	422
6	135	515	247 <sup>a</sup>	241	476	514	482	804
7	71	561	247	192	480 <sup>a</sup>	465	515 <sup>a</sup>	374
8	37	561	242	192	478	465	503	265
9	20	561	238	192	477	465	484	755
10	10	561	235	192	476	465	503	374

<sup>a</sup> Maximum returns with respect to number of years in nonhost crops.

ta(M) shows that the inequality in Eq. [13] holds locally at  $M = 7$ , but then there is another local maximum at  $M = 10$ . This result demonstrates why it is important to establish the neighborhood of the absolute maximum of the criterion function before using the adaptive procedures outlined in the paragraph following Eq. [13], where the focus is on using an estimated value of  $P$  as supplemental information for a final decision at planting time.

*Results for dynamic economic models:* A comparison of results from the steady-state economic model can be made with those from dynamic sequential decision models to evaluate the risk of using the simpler model. Dynamic programming (DP) is the most appropriate optimization model for this type of problem because results can be computed for an arbitrarily long planning horizon, and some of the decisions involve discrete choices instead of continuous variables (1). An important component of these models is the concept of *state variables*. A sufficient set of state variables completely summarizes the history of the decision process at a point in time, and an optimal decision rule exists that is a function of these state variables only. The model has a recursive structure such that the state variables are linked in adjacent time periods by a system of equations equal in number to the state variables, and the arguments of the functions in these state equations are the state and decision variables.

*a. Constant annual returns from nonhost crops:* The simplest case of nematode control is where an average return is used for the nonhost crop, population density is the only state variable, and there are only two decision alternatives (plant either the host or nonhost crop). The recursive equation of DP is

$$v_{n+1}(P) = \text{Max}[R_n(P) + wv_n(g(P)), R_a + wv_n(sP)], \quad (18)$$

where the new notation is defined as follows:

- $v_n(P)$  = present value of net returns from an  $n$ -stage process under an optimal decision rule when the initial state is  $P$
- $R_a$  = constant returns from the nonhost crop
- Max = the maximization operator over the two expressions in brackets.

The dynamic link that connects the two adjacent stages is the two functions embedded in  $v_n(\cdot)$  under the two choices available; note that these are the right-hand sides of the difference equations [1] and [2]. Numerical solution is accomplished by assigning zero to  $v_0(P)$  and then recursively solving Eq. [18] for  $v_n(P)$ ,  $n = 1, 2, \dots$ , for discrete values of  $P$ ; in this simple problem, the counting integers were used to measure number of eggs per 100 g of soil. Intuitively, the solution would seem to be structured so that, at small values of  $P$ , the decision is to plant the host crop and then after some threshold is reached, the optimal decision would be to plant the nonhost crop. This is the case, and the entire decision rule is characterized by that single switching point.

Results from computations under a variety of parametric situations, where Models 1 and 2 use different constants for net annual growth ( $g(P)$ ) of the nematode population under sugarbeet, 250.7 (Eq. [16] for the Sacramento Valley) and 501.4 (Eq. [15] for the Imperial Valley), respectively, are given in Table 3. Considering this magnitude of difference in the coefficients on the nematode growth functions for sugarbeet (a factor of two), economic thresholds in the dynamic optimization model are surprisingly close between Models 1 and 2, and would appear to have a small effect on the present value of returns if they were interchanged. The thresholds for the steady-state model are much more erratic and deviate considerably from the dynamically optimal values because only a few discrete values can be chosen for  $P$ , viz., the indirectly chosen steady-state values associated with  $M$  as it assumes one of

TABLE 3. Threshold population densities of *Heterodera schachtii* ( $P_M$ (\*), eggs/100 g soil) at which sugarbeets are planted following the optimal decision rules predicted by (i) dynamic optimization and (ii) a steady-state model. Annual returns of nonhost crops of \$210 and \$470, and a range of annual survival proportions ( $s$ ) were used. Thresholds are for the Sacramento Valley (Model 1) and the Imperial Valley (Model 2) in California.

s	Dynamic optimization model				Steady-state model			
	Model 1		Model 2		Model 1		Model 2	
	\$210	\$470	\$210	\$470	\$210	\$470	\$210	\$470
0.500	156	114	150	110	78	78	78	78
0.600	129	105	125	105	111	111	66	66
0.667	127	108	120	100	135	71	111	59
0.700	123	104	120	100	121	69	117	67
0.750	123	104	120	105	105	105	126	80

the counting integers,  $1, 2, \dots, M_{\max}$ . This limitation is apparently the reason for so few distinct threshold values appearing in Table 3 for the steady-state model. The most extreme case is where  $s = 0.5$  and all the thresholds are 78 eggs/100 g soil. In comparing the thresholds for the steady-state model, no exceptions are observed with respect to the threshold associated with the lower-return nonhost crops at least equaling that for the higher-return ones. Relatively low-valued nonhost crops would, in general, tend to increase the economic threshold on  $P$  at which sugarbeet is planted in order to plant the high-valued host crop more frequently.

Economic returns from the DP models were compared with the corresponding steady-state models by using amortized present value of returns, i.e., average annual returns weighted properly by the interest rate, but such comparisons are difficult because the present value of returns is dependent on the initial condition used, i.e., the initial state,  $P$ . The initial condition was calibrated to results from the steady-state model by recognizing that the nematode density at which the host crop is planted is transformed via  $g(P)$  to the initial density at which the first nonhost crop is planted. For example, in Table 2 with returns of \$210,  $P$  is equal to 135 at the time the host crop is planted, and this value is transformed to 1,531 by substitution into Eq. [16], which is the appropriate value to use as an initial condition for the

DP decision rule. Using this initial condition, the decision rule is applied sequentially until it is clear that the annuity calculated from present values of returns has converged to its limit for an infinite planning horizon. Then this value is compared with the annuity from the steady-state calculations, such as given in Table 2 for  $s = 0.667$ . In this example, the annuities are \$248.25 and \$247.44 under the optimal and steady-state derived decision rules, respectively. Surprisingly, the annuities are identical to the penny for  $s$  equal to 0.60, 0.70, and 0.75 when nonhost returns are \$210.

Similar results held for annual nonhost returns of \$470, the greatest differences across  $s$  being less than a dollar. A comparison of the threshold densities between the dynamic and steady-state models in Table 3 for Model 1 shows them to be close for either level of annual returns in nonhost crops, especially when considering the small impact that these differences would have on sugarbeet yields in Eq. [14]. Even the greatest difference in the table for  $s = 0.50$  is 156 versus 78, which would translate into 1 year more or less in nonhost crops with  $s$  so small.

Steady states under the optimal economic decision rules did not always take the simple form of a single cycle repeated indefinitely as in the steady-state models. Frequently, the total cycle would involve two or more subcycles embedded in the overall one. This phenomenon apparently

emanates from the possibility it provides for using the continuous variable  $P$  to compensate for the integer restriction on  $M$ . For example, with  $s$  and nonhost returns equal to 0.75 and \$470, respectively, the steady state is comprised of four subcycles of length 9, 9, 10, and 9. The optimal threshold density of *H. schachtii* is 104 eggs/100 g soil, and the levels of  $P$  at the beginning of the year that sugarbeet is planted are 100, 103, 78, and 94, while the resulting complete rotation takes 41 years.

*b. Irregular annual returns from nonhost crops:* A more complex DP model is required to deal with nonhost crops following an irregular pattern of returns such as the case reported in Table 2 from the steady-state decision rule. This is so because an additional state variable is required to identify the particular nonhost crop planted in the previous stage of the process, otherwise  $R_a$  would be undefined in Eq. [18] because it is no longer a constant independent of stages. The DP recursion formula for this case is given in Appendix B.

Since each nonhost crop planted the previous year in combination with *H. schachtii* egg density  $P$  defines a separate state in the decision model, the decision rule is characterized by three inequalities in  $P$ . For example when  $s = 0.667$ , sugarbeet is planted according to the following rule:

after tomato	$0 \leq P \leq 216$
after corn	$0 \leq P \leq 14$ and $93 \leq P \leq 109$
after wheat	never.

The gap between 14 and 93 after corn when sugarbeet is not planted seems counterintuitive, but under an optimal decision rule there can be many states that are not experienced after starting from an arbitrary initial condition. Under this decision rule and the particular initial conditions used for illustrative calculations of long-run returns, sugarbeet is planted only after tomato because the intervals for plant-

ing sugarbeet after corn in the above rule do not occur under the optimal policy. Since the decision rule involves returns from the last nonhost crop as well as those from the host crop at the current nematode density, the optimal policy will never include sugarbeet after wheat, which has the lowest value of the nonhost crops required in the rotation.

The same method as in the simpler DP model was used to make comparisons with the steady-state model. The initial value of  $P$  was chosen to mimic the steady-state model. Using the above decision rule with  $s = 0.667$ , the two annuities under the optimal versus steady-state rules were within a penny of each other. The greatest improvement using the DP decision rule was with  $s = 0.60$ , where the effective part of the rule is to plant sugarbeet after tomato if  $P \leq 185$ . This improved returns by about 1%.

Accuracy of the steady-state approximation also was explored with  $g(P)$  from the originally fitted equation in [15] as well as the one using all three data points (3); both project what appear to be rather large population values. The steady-state approximation procedures gave excellent results that were comparable to those reported above. Both of these alternative estimates of  $g(P)$  predict populations levels of *H. schachtii* about twice those predicted by the primary model, when the initial value is in the range of the economic thresholds. The magnitude of this larger population prediction on the optimal rotation is inversely related to the size of  $s$ , e.g., with  $s = 0.50$ , the rotation would be reduced by 1 year.

## DISCUSSION

These exploratory empirical results gave strong evidence that the steady-state approximation procedure will provide excellent practical decision rules that use dynamic economic thresholds. The most effective way to use these results depends on the reliability with which the actual population density of *H. schachtii*,  $P$ , can be es-

timated in the field at planting time for sugarbeet. If an accurate estimate can be obtained at a nominal cost, then the steady-state density  $P^*$  that maximizes profits should be treated as the critical point below which the host crop is planted. But if the uncertainty is great and(or) the cost is high in this measurement problem, the steady-state estimate of the optimal period in nonhost crops,  $M$ , should be used as additional information. If measurement of  $P$  were sufficiently unreliable, number of years since the host crop has been planted would be a better predictor of  $P$  than attempts to measure it directly, especially when used in conjunction with the level of crop loss in the most recently harvested host crop because this is an indicator of the beginning nematode density at the start of the period in nonhost crops.

If  $P$  can be estimated with sufficient accuracy, the economic trade-off equation in Eq. [13] could be used to great advantage as an aid in sequentially evaluating the pest population. The manager would be in a position to subjectively evaluate the weight that should be applied to the uncertain field measurement on  $P$  versus the unequivocal measure of years since the host crop has been planted. There would appear to be an opportunity to apply Bayesian decision theory to this situation, which would directly treat the uncertainty in the field estimation of  $P$  at planting time, but that opportunity is left for later research.

Annual returns in relation to length of rotation indicate that an error in the direction of too long a rotation is less costly than for a rotation too short. The costs become more asymmetric as the optimal rotation length increases because the host crop has less influence on average returns, and so an extra year in nonhost crops is less costly. Relatively large nonhost crop returns make the costs of too long a rotation less important because planting the nonhost crop is less of a sacrifice, but the penalty for too short a rotation has two dimensions in that sugarbeet yields are lowered and sugarbeets are planted more frequently.

Three quantitative relationships are required to use the steady-state model of nematode populations: (a) the host crop yield function  $R_h(P)$ , (b) the difference equation measuring population growth during the year in the host crop, and (c) the difference equation measuring population decline during years in nonhost crops. The latter two are the most challenging for empirical estimation because time series data are required, making the experiments subject to the year-to-year variations in climate and soil conditions (5-7).

The difficulty of measuring populations in field experiments with sufficient accuracy to permit precise statistical estimation is a serious problem. Statistical estimation bias in parameter estimators in regression models when the independent variable is measured with error is well known (18). If the nematode decline function is adequately described by the difference equation [1], it can be transformed to a function of time per se to avoid problems with measurement error in  $P_t$  on the right-hand side. This is essentially what Roberts et al. (13) did, although the unequal time intervals in the data would have required that this be done anyway. If the functional form of the growth function for  $g(p)$  in Eq. [7] is appropriate, the problem with measurement errors in the right-hand side variable is made easier because the equation is linear in logarithms, and can be estimated by well-known methods for linear models if information is available on the size of the measurement error in  $P_t$  relative to the total statistical error in the equation (18). Total statistical error is the sum of measurement error and the unexplained variation that would exist for the regression equation if there were no measurement error.

It is an over-simplification to conclude that the problem of measurement error in population densities could be avoided in the economic analysis by working directly with nematode densities as a function of time, as done by Ferris and Greco (9). Population densities as a function of time per

se are of limited use in decision models because these functions do not provide a structured recursive model for time-dated values of population density, which is needed for the steady-state approximation developed here, DP, or other systematic dynamic optimization models such as classical control theory (2). A linear first-order difference equation, such as the homogeneous case in Eq. [1], has a unique explicit functional form in time that would track the associated variable P, given one initial condition for it. The form of this equation is  $cs^t$ , where  $c$  is determined by an initial condition for P. The functional form used by Ferris and Greco (9) is  $at^b$ , which may or may not be the time domain solution to a *nonlinear* first-order difference equation. The authors used it with other empirical measures to evaluate many alternative 3-year crop sequences with respect to economic returns and terminal nematode densities. Their results provided considerable information on the economics of various crop rotation sequences but did not have the benefit of a formal sequential model.

When the entire research process is considered (conceptualization, experimental design, estimation of quantitative relationships, and economic analysis for application), there is considerable advantage in a formal recursive dynamic model of the biological system. The formal structure clarifies the interaction of components and their changes through time, as well as revealing critically needed data. It also focuses attention on the nature of the experiments required to provide the data, and whether it is feasible to pursue some of the quantification problems.

APPENDIX A: Derivation of M

The inequality in Eq. [13] that gives the necessary conditions for M to be the economically optimal period in nonhost crops is derived below. Since  $r$  in the numerator of the fraction in Eq. [12] is merely a multiplicative constant on the criterion to be maximized, we can replace it with unity to

simplify the algebra. Therefore, the task is to find the necessary conditions for

$$[A-1] \left[ \frac{\sum_{i=1}^M w^i R_{ai} + w^{M+1} R_h(f(M))}{(1 - w^{M+1})} \right] /$$

a maximum, which measures the present value of returns over an infinite planning horizon.

If M is to be the optimal integer that maximizes Eq. [A-1], then choose the smallest M that satisfies

$$\left[ \frac{\sum_{i=1}^M w^i R_{ai} + w^{M+1} R_h(f(M))}{(1 - w^{M+1})} \right] /$$

$$\geq \left[ \frac{\sum_{i=1}^{M+1} w^i R_{ai} + w^{M+2} R_h(f(M+1))}{(1 - w^{M+2})} \right] /$$

Multiply by  $(1 - w^{M+1})(1 - w^{M+2})$  to get

$$(1 - w^{M+2}) \left[ \sum_{i=1}^M w^i R_{ai} + w^{M+1} R_h(f(M)) \right]$$

$$\geq (1 - w^{M+1}) \left[ \sum_{i=1}^{M+1} w^i R_{ai} + w^{M+2} R_h(f(M+1)) \right].$$

We have

$$\sum_{i=1}^M w^i R_{ai}$$

on both sides of the above inequality, so these terms vanish, and the remainder can be written as

$$-w^{M+2} \sum_{i=1}^M w^i R_{ai}$$

$$+ (1 - w^{M+2})w^{M+1} R_h(f(M))$$



$$\geq w^{M+1}R_{a,M+1} - w^{M+1} \sum_{i=1}^{M+1} w^i R_{ai} + (1 - w^{M+1})w^{M+2}R_h(f(M + 1)).$$

Then divide this by  $w^{M+2}$ ,

$$- \sum_{i=1}^M w^i R_{ai} + (1 - w^{M+2})(1 + r)R_h(f(M)) \geq (1 + r)R_{a,M+1} - (1 + r) \sum_{i=1}^{M+1} w^i R_{ai} + (1 - w^{M+1})R_h(f(M + 1)).$$

The first term on the left side,

$$- \sum_{i=1}^M w^i R_{ai},$$

has a matching term on the right side, and thus vanishes. We can then write the inequality as

$$[r + (1 - w^{M+1})]R_h(f(M)) \geq (1 + r)R_{a,M+1} - (1 + r)w^{M+1}R_{a,M+1} - r \sum_{i=1}^M w^i R_{ai} + (1 - w^{M+1})R_h(f(M + 1)).$$

Dividing by  $1 - w^{M+1}$  and transposing terms, we obtain

$$[A-2] \quad R_h(f(M + 1)) - R_h(f(M)) + (1 + r)R_{a,M+1} \leq \frac{r}{1 - w^{M+1}} [R_h(f(M)) + \sum_{i=1}^M w^i R_{ai}].$$

This is one useful version of the necessary conditions for  $M$ , the economically optimal period in nonhost crops, but a more intuitive form can be derived.

First, note that  $1/(1 - w^n) = 1 + w^n/(1 - w^n)$ , which is applied to the above term,

$$\frac{r}{1 - w^{M+1}} R_h(f(M)) = rR_h(f(M)) + \frac{r}{1 - w^{M+1}} w^{M+1}R_h(f(M)).$$

Then the term  $rR_h(f(M))$  is transposed to the left side of the equation to yield the revised necessary condition,

$$[A-3] \quad [R_h(f(M + 1)) - (1 + r)R_h(f(M))] + (1 + r)R_{a,M+1} \leq \frac{r}{1 - (1 + r)^{-(M+1)}} \left[ \sum_{i=1}^M w^i R_{ai} + w^{M+1}R_h(f(M)) \right],$$

which is Eq. [13] in the text before  $f(M)$  and  $f(M + 1)$  were replaced with corresponding levels of the nematode population,  $P_e$  and  $dP_e$ , respectively.

#### APPENDIX B: SUMMARY OF THE DYNAMIC PROGRAMMING MODEL.

This appendix summarizes the essential aspects of the dynamic programming model used in the application in the text.

*State Variables:*

$P$  = nematode density

$L$  = subscript of last crop grown (last year's crop)

$L$	Crop Name
1	tomato
2	corn
3	wheat
4	sugarbeet (host crop)

*DP Recursion Structure:*

$$v_{n+1}(P, 1) = \text{Max}[R_h(P) + wv_n(g(P), 4), R_a(2) + wv_n(sP, 2)]$$

$$v_{n+1}(P, 2) = \text{Max}[R_h(P) + wv_n(g(P), 4), R_a(3) + wv_n(sP, 3)]$$

$$v_{n+1}(P, 3) = \text{Max}[R_h(P) + wv_n(g(P), 4), R_a(1) + wv_n(sP, 1)]$$

$$v_{n+1}(P, 4) = R_a(1) + wv_n(sP, 1)$$

The simplification for state  $(P, 4)$  occurs because planting the host crop would imply 2 consecutive years of the host crop.

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